Power and Exponential Laws.

Teacher Notes

Introduction

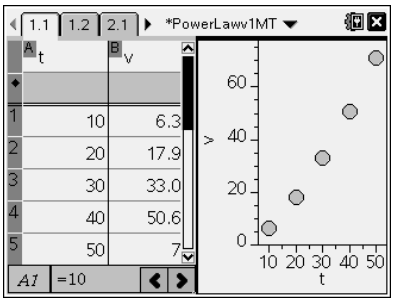
This activity involves the use of a straight-line graph to confirm relationships of the form $y = ax^b$ and $y=a.b^x$. Students must be able to model mathematically situations involving power or exponential functions eg. from experimental data they may be required to draw a graph of log y against log x and to deduce values of a and b such that $y = ax^b$.

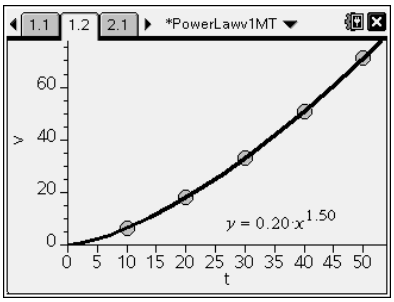
TI-Nspire is first used to draw a quick graph of experimental data to establish that there is some kind of connection between the variables. Then the Regression tools in the Analyze menu are used to determine what the relationship is.

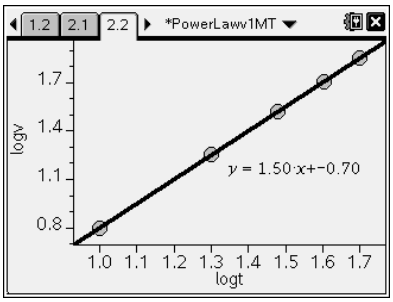
TI-Nspire is then used to graph $\log_{10}$ y against $\log_{10}$ x and show that there is a linear connection. This leads to how the equation can be found using mathematical methods without using the advanced TI-Nspire analysis facilities.

There are two parts to the activity:-

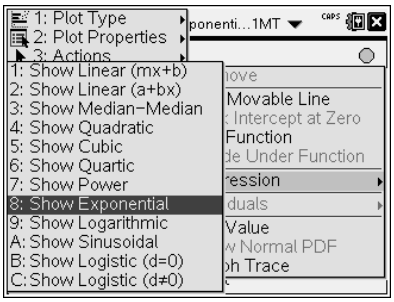
- $\log_{10}$ y against $\log_{10}$ x (Power Law)

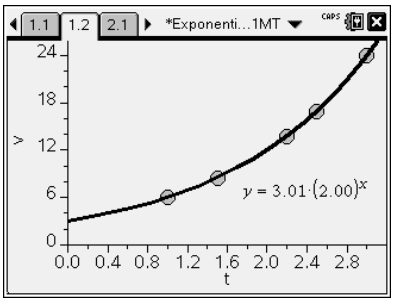


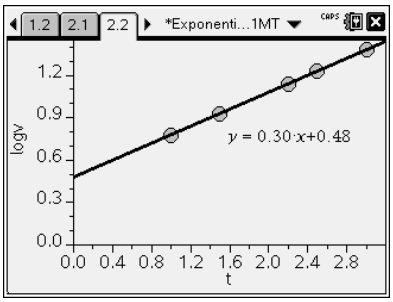




- $\log_{10}$ y against x (Exponential Law)







Resources

The TI-Nspire document PowerLaw deals with relationships of the form $y = ax^b$ and graphs $\log_{10}$ y against $\log_{10}$ x.
The TI-Nspire document ExponentialLaw deals with relationships of the form $y=a.b^x$ and graphs $\log_{10}$ y against x.
There are two PowerPoint presentations to support students/teachers who are unfamiliar with the TI-Nspire, one for each activity.
There is a worksheet for both activities as well as full solutions below.

Skills required

It is assumed that students will be able to carry out the following basic TI-Nspire processes.

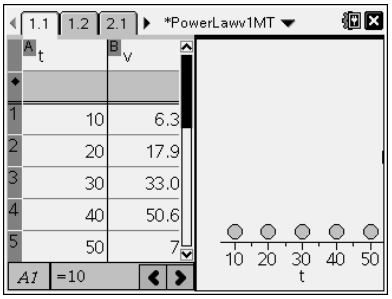
- ✓ Open and save a new tns document.
- ✓ Move from one page to another.
- ✓ Use menus to select commands.
- ✓ In a Data & Statistics page change variables on the axes

Other more unusual techniques are described in full in the teacher's notes.

The Power Law activity, plotting $log_{10} y$ against $log_{10} x$.

The TI-Nspire document PowerLawv1.tns is divided into 10 examples.

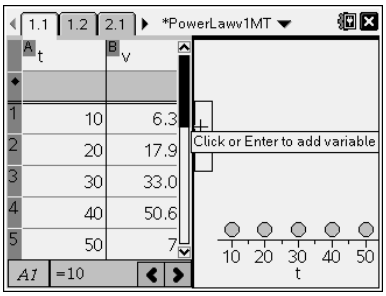
Example 1. A worked example



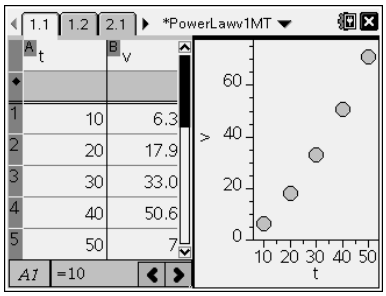
Page 1.1 shows a table of values for t and v.

To use quick graph to show a connection between the variables press:
(menu) (3) Data (6) Quick Graph

Move the cursor to the middle of the y-axis.
Press (enter)

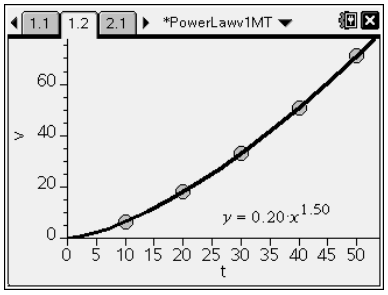


Choose v (enter)

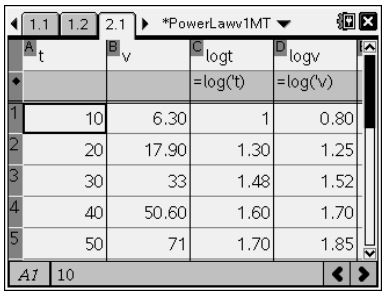


Move to page 1.2 (ctrl)▶

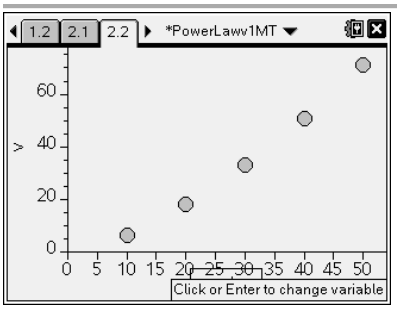
Find the equation connecting the variables:
(menu) (4) Analyse (6) Regression (7) Show Power

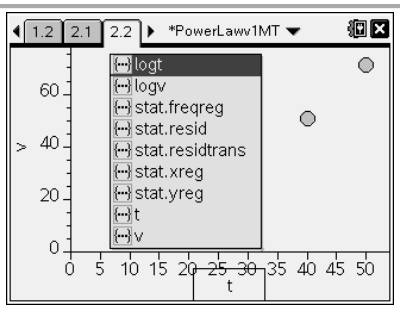


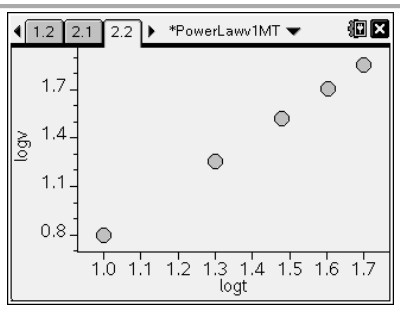
Move to page 2.1 where a table of values for $\log_{10}$ t and $\log_{10}$ v has been constructed.



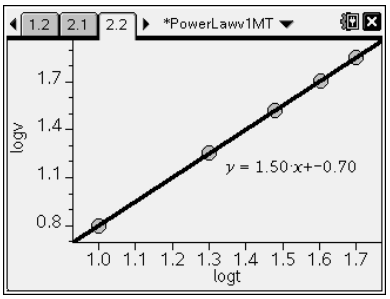
Move to page 2.2. Change the variables on the axes to $\log_{10}$ t and $\log_{10}$ v.







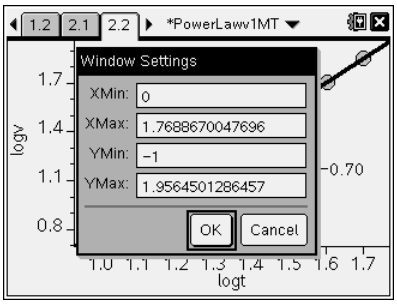
Find the equation of the straight line.
(menu) (4) Analyse (6) Regression
(1) Show Linear(mx+b)

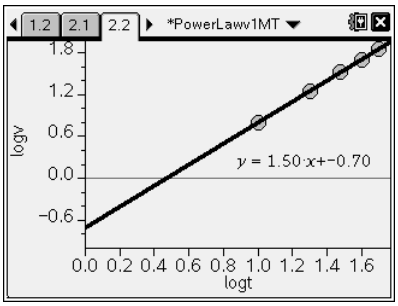


Change the window settings to see the y-intercept.

(menu) (5) Window/Zoom
(1) Window Settings

Change XMin to 0
and YMin to -1
using tab to navigate.





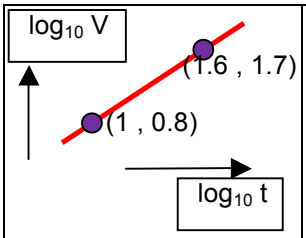
Now introduce the mathematical strategies required to find the equations.
Students complete the Power Law Worksheet Example 1 along with the teacher.
The completed solution for Example 1 is shown below.

POWER LAW WORKSHEET

Example 1

t	10	20	30	40	50
V	6.3	17.9	33.0	50.6	71.0

$\log_{10} t$	1.00	1.30	1.48	1.60	1.70
$\log_{10} V$	0.80	1.25	1.52	1.70	1.85



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.7 - 0.8}{1.6 - 1} = \frac{0.9}{0.6} = 1.5$$

Equation of Straight Line

$$Y = mX + c$$

$$\log_{10} v = 1.5 \log_{10} t + c$$

Find y intercept

(1 , 0.8) lies on the line.

$0.8 = 1.5 \times 1 + c$

$0.8 - 1.5 = c$

$c = -0.7$

$$\log_{10} V = 1.5 \log_{10} t - 0.7$$

Equation of Power Function

$$\log_{10} V = 1.5 \log_{10} t - 0.7$$

$\log_{10} (?) = -0.7$

$(?) = 10^{-0.7}$

$$\log_{10} V = \log_{10} t^{1.5} + \log_{10} (10^{-0.7})$$

$$\log_{10} V = \log_{10} t^{1.5} + \log_{10} (0.2)$$

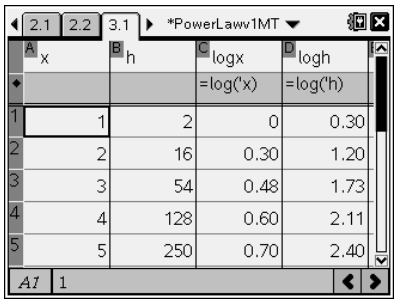
$$\log_{10} V = \log_{10} (t^{1.5} \times 0.2)$$

$$V = 0.2\, t^{1.5}$$

Example 2

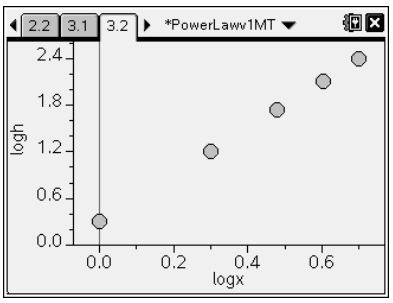
This is another worked example. This time the equation of the straight line is worked out first using mathematics and it is then checked using the Regression tool of the handheld.

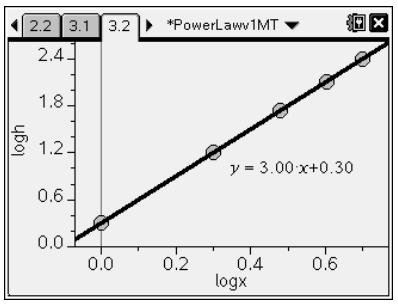
Move to page 3.1 where a table of values for x,h,$\log_{10}$ x and $\log_{10}$ h has been constructed.



To check the equation of the straight line move to page 3.2

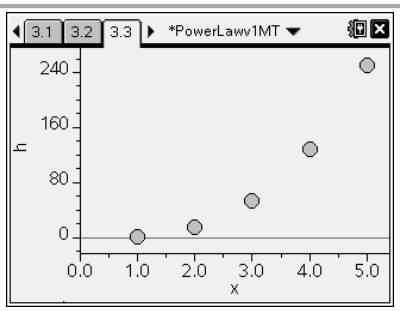
menu (4) Analyse
(6) Regression
(1) Show Linear(mx+b)

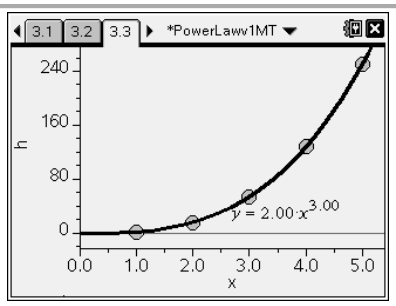




To check the equation connecting the variables move to page 3.3

(menu) (4) Analyse
(6) Regression
(7) Show Power

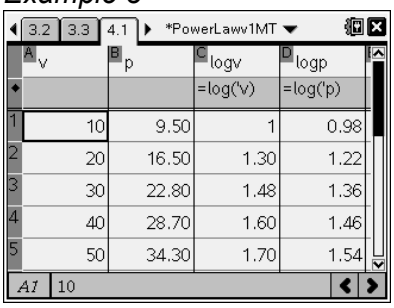


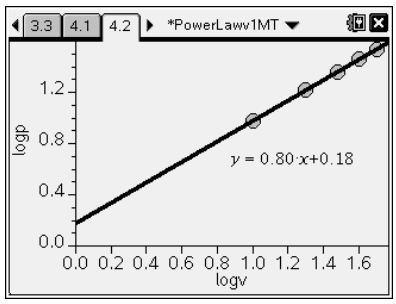


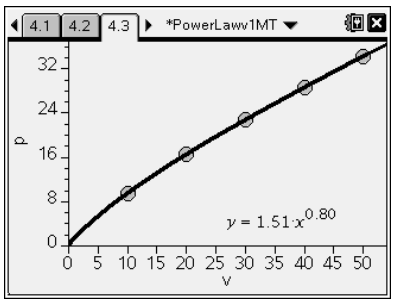
Examples 3 and 4

Students then work on Examples 3 and 4, finding the equations first using mathematics then using the handheld to check the answers.

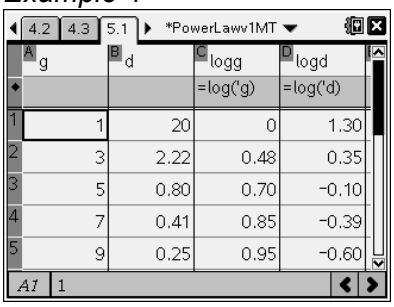
Example 3

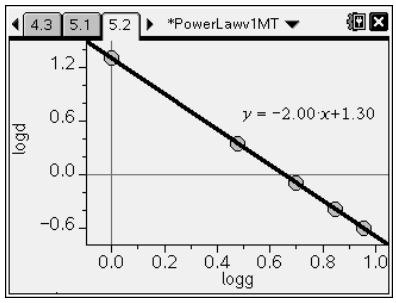


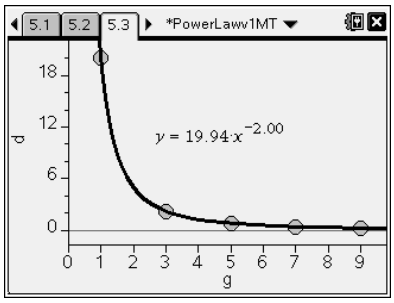




Example 4



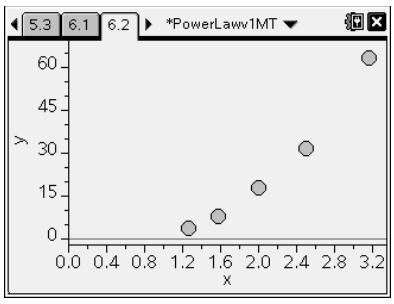




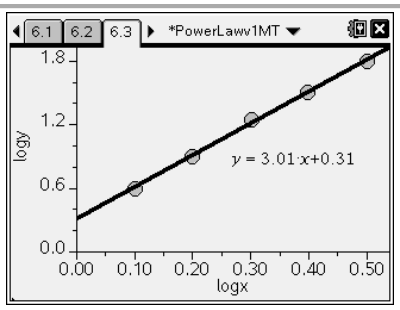
Examples 5 to 10.

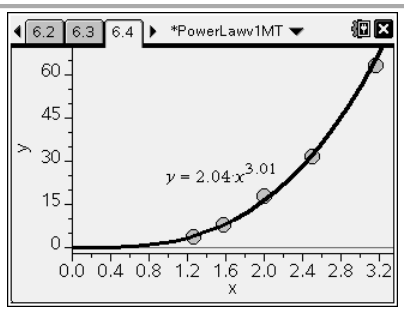
Page 1 of each example shows the table of values.

Students move to page 2 to see that there is a connection between the variables and find the equations using mathematics.



These equations are then checked on pages 3 and 4.

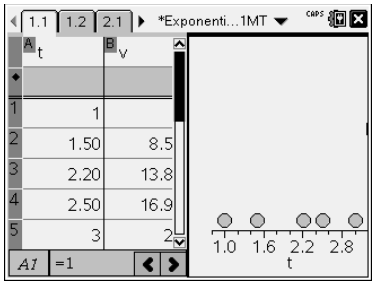




The Exponential Law activity, plotting $log_{10} y$ against x.

The TI-Nspire document ExponentialLawv1.tns is divided into 7 Examples.

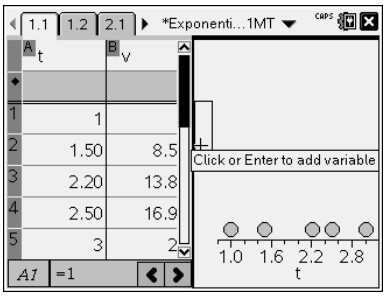
Example 1. A worked example



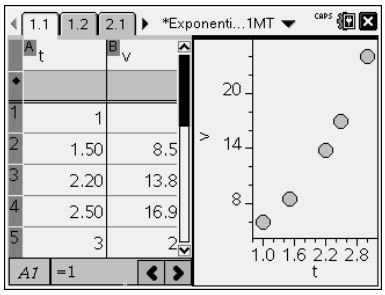
Page 1.1 shows a table of values for t and v.

To use quick graph to show a connection between the variables press:
(menu) (3) Data (6) Quick Graph

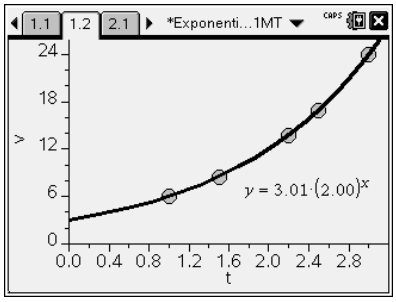
Move the cursor to the middle of the y-axis.
Press (enter)



Choose v (enter)



Move to page 1.2 (ctrl)▶



Find the equation connecting the variables
(menu) (4) Analyse
(6) Regression
(8) Show Exponential

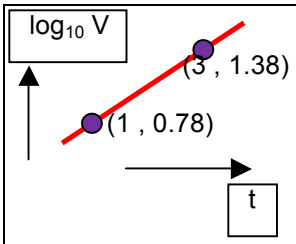
Mathematics can now be used to find the equation.
Students complete the Exponential Law Worksheet Example 1 along with the teacher.
The completed solution for Example 1 is shown on the next page.

EXPONENTIAL LAW WORKSHEET

Example 1

t	1	1.5	2.2	2.5	3
V	6	8.5	13.8	16.9	24

t	1	1.5	2.2	2.5	3
$\log_{10} V$	0.78	0.93	1.14	1.23	1.38



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.38 - 0.78}{3 - 1} = \frac{0.6}{2} = 0.3$$

Equation of Straight Line

$$Y = mX + c$$

$$\log_{10} V = 0.3\ t + c$$

Find y intercept

(1 , 0.78) lies on the line.

$0.78 = 0.3 \times 1 + c$

$0.78 - 0.3 = c$

$c = 0.48$

$$\log_{10} V = 0.3\ t + 0.48$$

Equation of Exponential Function

$$\log_{10} V = 0.3\ t + 0.48$$

$\log_{10} (?) = 0.3$ $\quad (?) = 10^{0.3}$

$\log_{10} (?) = 0.48$ $\quad (?) = 10^{0.48}$

$$\log_{10} V = \log_{10} (10^{0.3})\ t + \log_{10} (10^{0.48})$$

$$\log_{10} V = \log_{10} (2.0)\ t + \log_{10} (3.0)$$

$$\log_{10} V = t \log_{10} (2.0) + \log_{10} (3.0)$$

$$\log_{10} V = \log_{10} (2.0)^{t.} + \log_{10} (3.0)$$

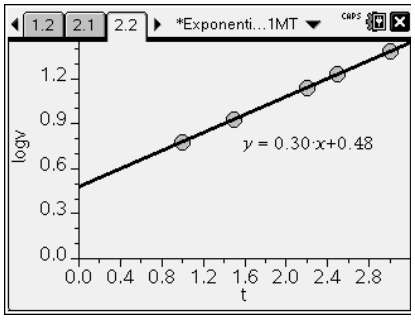
$$\log_{10} V = \log_{10} ((2.0)^t \times 3.0)$$

$$V = 3.0\ (2.0)^t$$

Note that the gradient here needs to be correct to 2 d.p. as all log values have been rounded to this degree of accuracy. Examples 2 to 7 need to be rounded.

Move to page 2.2 (ctrl)▶

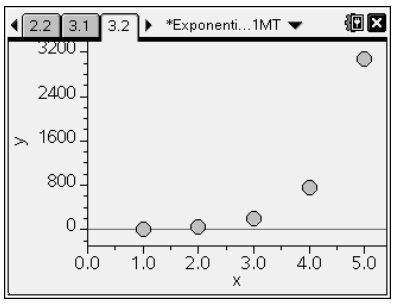
Check the equation of the straight line.
(menu) (4) Analyse
(6) Regression
(1) Show Linear(mx+b)



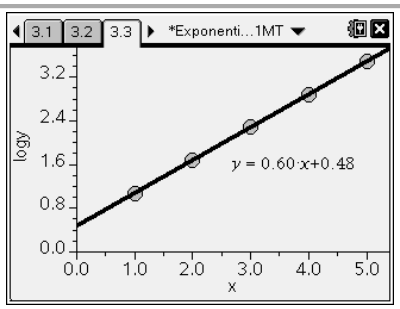
Examples 2 to 7.

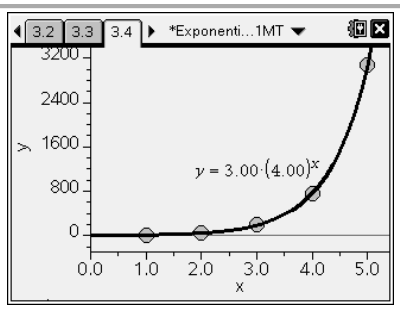
Page 1 of each example shows the table of values.

Students move to page 2 to see that there is a connection between the variables and find the equations using mathematics.



These equations are then checked on pages 3 and 4.



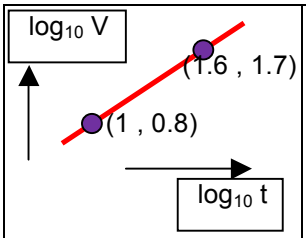


POWER LAW WORKSHEET SOLUTIONS

Example 1

t	10	20	30	40	50
V	6.3	17.9	33.0	50.6	71.0

$\log_{10} t$	1.00	1.30	1.48	1.60	1.70
$\log_{10} V$	0.80	1.25	1.52	1.70	1.85



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.7 - 0.8}{1.6 - 1} = \frac{0.9}{0.6} = 1.5$$

Equation of Straight Line

$Y = mX + c$

$\log_{10} v = 1.5 \log_{10} t + c$

Find y intercept

(1 , 0.8) lies on the line.

$0.8 = 1.5 \times 1 + c$

$0.8 - 1.5 = c$

$c = -0.7$

$\log_{10} V = 1.5 \log_{10} t - 0.7$

Equation of Power Function

$\log_{10} V = 1.5 \log_{10} t - 0.7$

$\log_{10} (?) = -0.7$

$(?) = 10^{-0.7}$

$\log_{10} V = \log_{10} t^{1.5} + \log_{10} (10^{-0.7})$

$\log_{10} V = \log_{10} t^{1.5} + \log_{10} (0.2)$

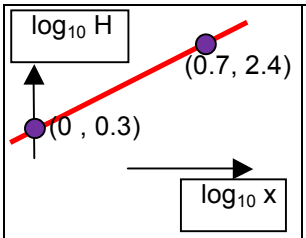
$\log_{10} V = \log_{10} (t^{1.5} \times 0.2)$

$V = 0.2\, t^{1.5}$

Example 2

x	1	2	3	4	5
H	2	16	54	128	250

$\log_{10} x$	0	0.30	0.48	0.60	0.70
$\log_{10} H$	0.30	1.20	1.73	2.11	2.40



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.4 - 0.3}{0.7 - 0} = \frac{2.1}{0.7} = 3$$

Equation of Straight Line

$Y = mX + c$

$\log_{10} H = 3 \log_{10} x + c$

Find y intercept

(0 , 0.3) lies on the line.

$c = 0.3$

$\log_{10} H = 3 \log_{10} x + 0.3$

Equation of Power Function

$\log_{10} H = 3 \log_{10} x + 0.3$

$\log_{10} (?) = 0.3$

$(?) = 10^{0.3}$

$\log_{10} H = \log_{10} x^3 + \log_{10} (10^{0.3})$

$\log_{10} H = \log_{10} x^3 + \log_{10} (2)$

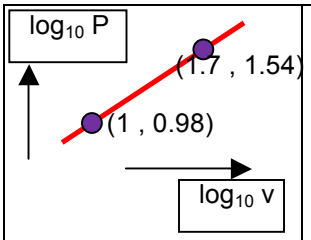
$\log_{10} H = \log_{10} (x^3 \times 2)$

$H = 2\, x^3$

Example 3

v	10	20	30	40	50
P	9.5	16.5	22.8	28.7	34.3

$\log_{10} v$	1.00	1.30	1.48	1.60	1.70
$\log_{10} P$	0.98	1.22	1.36	1.46	1.54



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.54 - 0.98}{1.7 - 1} = \frac{0.56}{0.7} = 0.8$$

Equation of Straight Line

$Y = mX + c$

$\log_{10} P = 0.8 \log_{10} v + c$

Find y intercept

(1,0.98) lies on the line.

$0.98 = 0.8 \times 1 + c$

$0.98 - 0.8 = c$

$c = 0.18$

$\log_{10} P = 0.8 \log_{10} v + 0.18$

Equation of Power Function

$\log_{10} P = 0.8 \log_{10} v + 0.18$

$\log_{10} (?) = 0.18$

$(?) = 10^{0.18}$

$\log_{10} P = \log_{10} v^{0.8} + \log_{10} (10^{0.18})$

$\log_{10} P = \log_{10} v^{0.8} + \log_{10} (1.5)$

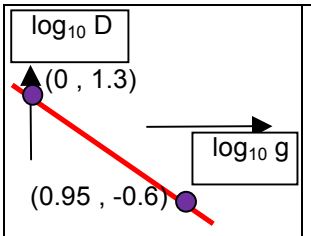
$\log_{10} P = \log_{10} (v^{0.8} \times 1.5)$

$P = 1.5\, v^{0.8}$

Example 4

g	1	3	5	7	9
D	20	2.22	0.80	0.41	0.25

$\log_{10} g$	0	0.48	0.70	0.85	0.95
$\log_{10} D$	1.30	0.35	-0.10	-0.39	-0.60



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.3 - (-0.6)}{0 - 0.95} = \frac{1.9}{-0.95} = -2$$

Equation of Straight Line

$Y = mX + c$

$\log_{10} D = -2 \log_{10} g + c$

Find y intercept

(0 , 1.3) lies on the line.

$c = 1.3$

$\log_{10} D = -2 \log_{10} g + 1.3$

Equation of Power Function

$\log_{10} D = -2 \log_{10} g + 1.3$

$\log_{10} (?) = 1.3$

$(?) = 10^{1.3}$

$\log_{10} D = \log_{10} g^{-2} + \log_{10} (10^{1.3})$

$\log_{10} D = \log_{10} g^{-2} + \log_{10} (20)$

$\log_{10} D = \log_{10} (g^{-2} \times 20)$

$D = 20\, g^{-2}$

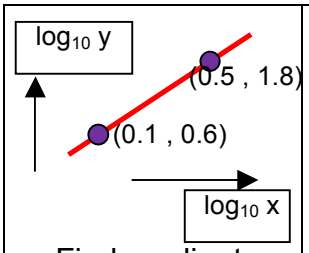
Examples 5 to 10

For each example
(i) show that the formula connecting y and x is of the form $y = kx^n$ (on page 2 of handheld).
(ii) find the value of k and n, and state the formula that connects x and y.
Check the equation of the straight line (page 3) and the power function (page 4) on the handheld.

5).

x	1.26	1.58	2.00	2.50	3.16
y	3.98	7.94	17.78	31.60	63.10

$\log_{10} x$	0.10	Assume first and last point lie on line of best fit.	0.50
$\log_{10} y$	0.60		1.80



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.8 - 0.6}{0.5 - 0.1} = \frac{1.2}{0.4} = 3$$

Equation of Straight Line

$Y = mX + c$

$\log_{10} y = 3 \log_{10} x + c$

Find y intercept

(0.1 , 0.6) lies on the line.
$0.6 = 3 \times 0.1 + c$
$0.6 - 0.3 = c$
$c = 0.3$

$\log_{10} y = 3 \log_{10} x + 0.3$

Equation of Power Function

$\log_{10} y = 3 \log_{10} x + 0.3$

$\log_{10} (?) = 0.3$

$(?) = 10^{0.3}$

$\log_{10} y = \log_{10} x^3 + \log_{10} (10^{0.3})$

$\log_{10} y = \log_{10} x^3 + \log_{10} (2)$

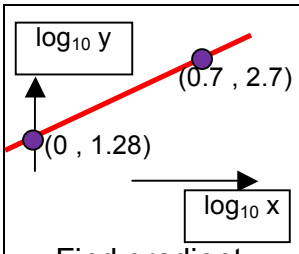
$\log_{10} y = \log_{10} (x^3 \times 2)$

$y = 2 x^3$

6).

x	1	2	3	4	5
y	19	80	177	316	500

$\log_{10} x$	0	Assume first and last point lie on line of best fit.	0.70
$\log_{10} y$	1.28		2.70



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.7 - 1.28}{0.7 - 0} = \frac{1.42}{0.7} = 2.0 \text{ (1d.p.)}$$

Equation of Straight Line

$Y = mX + c$

$\log_{10} y = 2 \log_{10} x + c$

Find y intercept

(0 , 1.28) lies on the line.
$c = 1.28$

$\log_{10} y = 2 \log_{10} x + 1.28$

Equation of Power Function

$\log_{10} y = 2 \log_{10} x + 1.28$

$\log_{10} (?) = 1.28$

$(?) = 10^{1.28}$

$\log_{10} y = \log_{10} x^2 + \log_{10} (10^{1.28})$

$\log_{10} y = \log_{10} x^2 + \log_{10} (19.1)$

$\log_{10} y = \log_{10} (x^2 \times 19.1)$

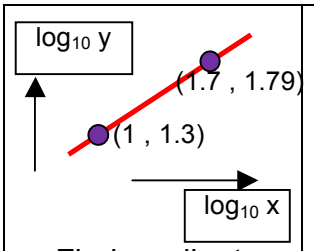
$y = 19.1 x^2$

handheld answer $19.2 x^2$

7).

x	10	20	30	40	50
y	20	32.6	43.3	52.9	61.8

$\log_{10} x$	1	Assume first and last point lie on line of best fit.	1.70
$\log_{10} y$	1.3		1.79



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.79 - 1.3}{1.7 - 1} = \frac{0.49}{0.7} = 0.7$$

Equation of Straight Line

$Y = mX + c$

$\log_{10} y = 0.7 \log_{10} x + c$

Find y intercept

(1 , 1.3) lies on the line.

$1.3 = 0.7 \times 1 + c$

$1.3 - 0\ 7 = c$

$c = 0.6$

$\log_{10} y = 0.7 \log_{10} x + 0.6$

Equation of Power Function

$\log_{10} y = 0.7 \log_{10} x + 0.6$

$\log_{10} (?) = 0.6$

$(?) = 10^{0.6}$

$\log_{10} y = \log_{10} x^{0.7} + \log_{10} (10^{0.6})$

$\log_{10} y = \log_{10} x^{0.7} + \log_{10} (4.0)$

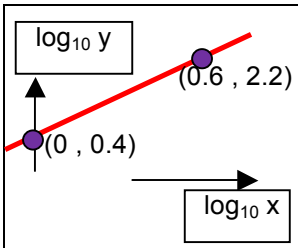
$\log_{10} y = \log_{10} (x^{0.7} \times 4)$

$y = 4 x^{0.7}$

8).

x	1	1.5	2	3	4
y	2.50	8.42	20	67.50	160

$\log_{10} x$	0	Assume first and last point lie on line of best fit.	0.60
$\log_{10} y$	0.40		2.20



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2.2 - 0.4}{0.6 - 0} = \frac{1.8}{0.6} = 3$$

Equation of Straight Line

$Y = mX + c$

$\log_{10} y = 3 \log_{10} x + c$

Find y intercept

(0 , 0.4) lies on the line.

$c = 0.4$

$\log_{10} y = 3 \log_{10} x + 0.4$

Equation of Power Function

$\log_{10} y = 3 \log_{10} x + 0.4$

$\log_{10} (?) = 0.4$

$(?) = 10^{0.4}$

$\log_{10} y = \log_{10} x^3 + \log_{10} (10^{0.4})$

$\log_{10} y = \log_{10} x^3 + \log_{10} (2.5)$

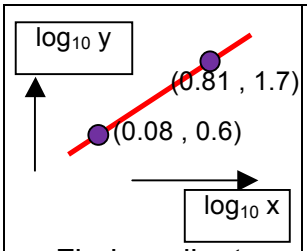
$\log_{10} y = \log_{10} (x^3 \times 2.5)$

$y = 2.5 x^3$

9).

x	1.2	3.1	4.2	5.5	6.5
y	3.94	16.37	25.80	38.70	49.70

$\log_{10} x$	0.08	Assume first and last point lie on line of best fit.	0.81
$\log_{10} y$	0.60		1.70



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1.7 - 0.6}{0.81 - 0.08}$$

$$= \frac{1.1}{0.73} = 1.5 \text{ (1d.p.)}$$

Equation of Straight Line

$$Y = mX + c$$

$$\log_{10} y = 1.5 \log_{10} x + c$$

Find y intercept

(0.08 , 0.6) lies on the line.

$0.6 = 1.5 \times 0.08 + c$

$0.6 - 0\ 12 = c$

$c = 0.48$

$$\log_{10} y = 1.5 \log_{10} x + 0.48$$

Equation of Power Function

$$\log_{10} y = 1.5 \log_{10} x + 0.48$$

$\log_{10}(?) = 0.48$

$(?) = 10^{0.48}$

$$\log_{10} y = \log_{10} x^{1.5} + \log_{10} (10^{0.48})$$

$$\log_{10} y = \log_{10} x^{1.5} + \log_{10} (3.0)$$

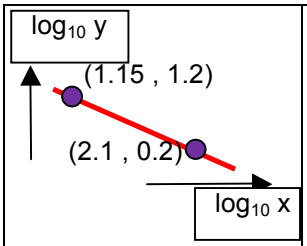
$$\log_{10} y = \log_{10} (x^{1.5} \times 3)$$

$$y = 3 x^{1.5}$$

10).

x	14.1	28.2	63.1	126
y	15.90	6.31	3.16	1.58

$\log_{10} x$	1.15	Assume first and last point lie on line of best fit.	2.10
$\log_{10} y$	1.20		0.20



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1.2 - 0.2}{1.15 - 2.1}$$

$$= \frac{1}{-0.95} = -1.1 \text{ (1d.p.)}$$

Equation of Straight Line

$$Y = mX + c$$

$$\log_{10} y = -1\ 1 \log_{10} x + c$$

Find y intercept

(1.15 , 1.2) lies on the line.

$1.2 = -1 \times 1.15 + c$

$1.2 + 1.15 = c$

$c = 2.35$

$$\log_{10} y = -1\ 1 \log_{10} x + 2.35$$

handheld answer $-1.03 \log_{10} + 2.35$

Equation of Power Function

$$\log_{10} y = -1\ 1 \log_{10} x + 2.35$$

$\log_{10}(?) = 2.35$

$(?) = 10^{2.35}$

$$\log_{10} y = \log_{10} x^{-1.1} + \log_{10} (10^{2.35})$$

$$\log_{10} y = \log_{10} x^{-1.1} + \log_{10} (224)$$

$$\log_{10} y = \log_{10} (x^{-1.1} \times 224)$$

$$y = 224 x^{-1.1}$$

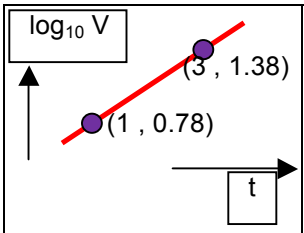
handheld answer $224 x^{-1.03}$

EXPONENTIAL LAW WORKSHEET SOLUTIONS

Example 1

t	1	1.5	2.2	2.5	3
V	6	8.5	13.8	16.9	24

t	1	1.5	2.2	2.5	3
$\log_{10} V$	0.78	0.93	1.14	1.23	1.38



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.38 - 0.78}{3 - 1} = \frac{0.6}{2} = 0.3$$

Equation of Straight Line

$$Y = mX + c$$

$$\log_{10} V = 0.3\ t + c$$

Find y intercept

(1 , 0.78) lies on the line.

$0.78 = 0.3 \times 1 + c$

$0.78 - 0.3 = c$

$c = 0.48$

$$\log_{10} V = 0.3\ t + 0.48$$

Equation of Exponential Function

$$\log_{10} V = 0.3\ t + 0.48$$

$\log_{10} (?) = 0.3$ $\quad (?) = 10^{0.3}$

$\log_{10} (?) = 0.48$ $\quad (?) = 10^{0.48}$

$$\log_{10} V = \log_{10} (10^{0.3})\ t + \log_{10} (10^{0.48})$$

$$\log_{10} V = \log_{10} (2.0)\ t + \log_{10} (3.0)$$

$$\log_{10} V = t \log_{10} (2.0) + \log_{10} (3.0)$$

$$\log_{10} V = \log_{10} (2.0)^{t.} + \log_{10} (3.0)$$

$$\log_{10} V = \log_{10} ((2.0)^t \times 3.0)$$

$$V = 3.0\ (2.0)^t$$

Examples 2 to 7

For each example:

(i) show that the formula connecting y and x is of the form $y = a.b^x$ (on page 2 of handheld).

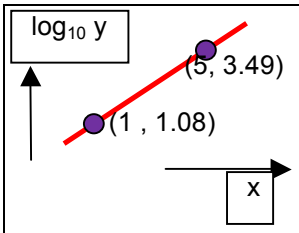
(ii) find the value of a and b, and state the formula that connects x and y.

Check the equation of the straight line (page 3) and the exponential function (page 4) on the handheld.

2).

x	1	2	3	4	5
y	12	48	192	768	3072

x	1	Assume first and last point lie on line of best fit.	5
$\log_{10} y$	1.08		3.49



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3.49 - 1.08}{5 - 1}$$

$$= \frac{2.41}{4} = 0.60 \text{ (2d.p.)}$$

Equation of Straight Line

$$Y = mX + c$$

$$\log_{10} y = 0.60\, x + c$$

Find y intercept

(1 , 1.08) lies on the line.

$1.08 = 0.60 \times 1 + c$

$1.08 - 0.6 = c$

$c = 0.48$

$$\log_{10} y = 0.60\, x + 0.48$$

Equation of Exponential Function

$$\log_{10} y = 0.60\, x + 0.48$$

$\log_{10} (?) = 0.60$ $\quad (?) = 10^{0.60}$

$\log_{10} (?) = 0.48$ $\quad (?) = 10^{0.48}$

$$\log_{10} y = \log_{10} (10^{0.60})\, x + \log_{10} (10^{0.48})$$

$$\log_{10} y = \log_{10} (4.0)\, x + \log_{10} (3.0)$$

$$\log_{10} y = x \log_{10} (4.0) + \log_{10} (3.0)$$

$$\log_{10} y = \log_{10} (4.0)^{x} + \log_{10} (3.0)$$

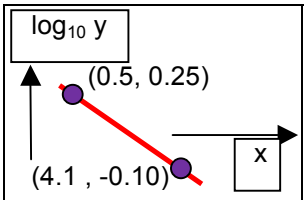
$$\log_{10} y = \log_{10} ((4.0)^{x} \times 3.0)$$

$$y = 3.0\, (4.0)^{x}$$

3).

x	0.5	1.2	3.8	4.1
y	1.79	1.53	0.86	0.80

x	0.5	Assume first and last point lie on line of best fit.	4.1
$\log_{10} y$	0.25		-0.10



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0.25 - (-0.1)}{0.5 - 4.1}$$

$$= \frac{0.35}{-3.6} = -0.10 \text{ (2d.p.)}$$

Equation of Straight Line

$$Y = mX + c$$

$$\log_{10} y = -0.10\, x + c$$

Find y intercept

(0.5 , 0.25) lies on the line.

$0.25 = = -0.10 \times 0.5 + c$

$0.25 + 0.05 = c$

$c = 0.30$

$$\log_{10} y = -0.10\, x + 0.30$$

Equation of Exponential Function

$$\log_{10} y = -0.10\, x + 0.30$$

$\log_{10} (?) = -0.10$ $\quad (?) = 10^{-0.10}$

$\log_{10} (?) = 0.30$ $\quad (?) = 10^{0.30}$

$$\log_{10} y = \log_{10} (10^{-0.10})\, x + \log_{10} (10^{0.30})$$

$$\log_{10} y = \log_{10} (0.8)\, x + \log_{10} (2.0)$$

$$\log_{10} y = x \log_{10} (0.8) + \log_{10} (2.0)$$

$$\log_{10} y = \log_{10} (0.8)^{x} + \log_{10} (2.0)$$

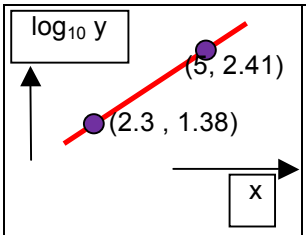
$$\log_{10} y = \log_{10} ((0.8)^{x} \times 2.0)$$

$$y = 2.0\, (0.8)^{x}$$

4).

x	2.3	3.2	4.6	5.0
y	23.97	52.70	179.52	254.80

x	2.3	Assume first and last point lie on line of best fit.	5.0
$\log_{10} y$	1.38		2.41



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2.41 - 1.38}{5 - 2.3}$$

$$= \frac{1.03}{2.7} = 0.38 \text{ (2d.p.)}$$

Equation of Straight Line

$$Y = mX + c$$

$$\log_{10} y = 0.38\, x + c$$

Find y intercept

(2.3 , 1.38) lies on the line.

$1.38 = 0.38 \times 2.3 + c$

$1.38 - 0.874 = c$

$c = 0.51$ (2d.p.)

$$\log_{10} y = 0.38\, x + 0.51$$

Equation of Exponential Function

$$\log_{10} y = 0.38\, x + 0.51$$

$\log_{10}(?) = 0.38$ $\quad (?) = 10^{0.38}$

$\log_{10}(?) = 0.51$ $\quad (?) = 10^{0.51}$

$$\log_{10} y = \log_{10}(10^{0.38})\, x + \log_{10}(10^{0.51})$$

$$\log_{10} y = \log_{10}(2.4)\, x + \log_{10}(3.2)$$

$$\log_{10} y = x \log_{10}(2.4) + \log_{10}(3.2)$$

$$\log_{10} y = \log_{10}(2.4)^{x} + \log_{10}(3.2)$$

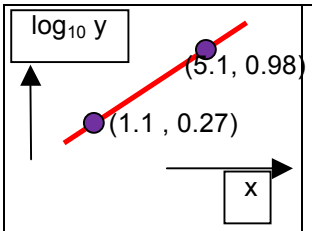
$$\log_{10} y = \log_{10}((2.4)^x \times 3.2)$$

$$y = 3.2\,(2.4)^x$$

5).

x	1.1	2.3	3.0	4.2	5.1
y	1.87	3.05	4.05	6.59	9.49

x	1.1	Assume first and last point lie on line of best fit.	5.1
$\log_{10} y$	0.27		0.98



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0.98 - 0.27}{5.1 - 1.1}$$

$$= \frac{0.71}{4} = 0.18 \text{ (2d.p.)}$$

Equation of Straight Line

$$Y = mX + c$$

$$\log_{10} y = 0.18\, x + c$$

Find y intercept

(1.1 , 0.27) lies on the line.

$0.27 = 0.18 \times 1.1 + c$

$0.27 - 0.198 = c$

$c = 0.07$ (2d.p.)

$$\log_{10} y = 0.18\, x + 0.07$$

handheld gives y intercept as 0.08

Equation of Exponential Function

$$\log_{10} y = 0.18\, x + 0.07$$

$\log_{10}(?) = 0.18$ $\quad (?) = 10^{0.18}$

$\log_{10}(?) = 0.07$ $\quad (?) = 10^{0.07}$

$$\log_{10} y = \log_{10}(10^{0.18})\, x + \log_{10}(10^{0.07})$$

$$\log_{10} y = \log_{10}(1.5)\, x + \log_{10}(1.2)$$

$$\log_{10} y = x \log_{10}(1.5) + \log_{10}(1.2)$$

$$\log_{10} y = \log_{10}(1.5)^{x} + \log_{10}(1.2)$$

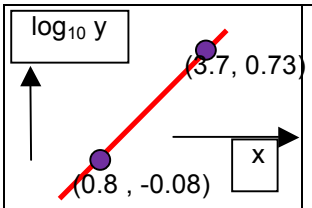
$$\log_{10} y = \log_{10}((1.5)^x \times 1.2)$$

$$y = 1.2\,(1.5)^x$$

6).

x	0.8	1.3	2.6	3.7
y	0.84	1.15	2.65	5.37

x	0.8	Assume first and last point lie on line of best fit	3.7
$\log_{10} y$	-0.08		0.73



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0.73 - (-0.08)}{3.7 - 0.8}$$

$$= \frac{0.81}{2.9} = 0.28 \text{ (2d.p.)}$$

Equation of Straight Line

$$Y = mX + c$$

$$\log_{10} y = 0.28\,x + c$$

Find y intercept

(0.8 , -0.08) lies on the line.

$-0.08 = 0.28 \times 0.8 + c$

$-0.08 - 0.224 = c$

$c = -0.30$ (2d.p.)

$$\log_{10} y = 0.28\,x - 0.30$$

Equation of Exponential Function

$$\log_{10} y = 0.28\,x - 0.30$$

$\log_{10}(?) = 0.28$ $\quad (?) = 10^{0.28}$

$\log_{10}(?) = -0.3$ $\quad (?) = 10^{-0.3}$

$$\log_{10} y = \log_{10}(10^{0.28})\,x + \log_{10}(10^{-0.3})$$

$$\log_{10} y = \log_{10}(1.9)\,x + \log_{10}(0.5)$$

$$\log_{10} y = x \log_{10}(1.9) + \log_{10}(0.5)$$

$$\log_{10} y = \log_{10}(1.9)^{x} + \log_{10}(0.5)$$

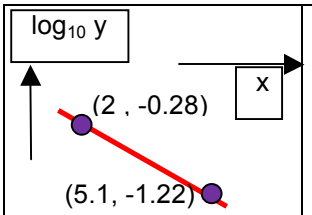
$$\log_{10} y = \log_{10}((1.9)^{x} \times 0.5)$$

$$y = 0.5\,(1.9)^{x}$$

7).

x	2.0	3.1	3.8	4.4	5.1
y	0.53	0.24	0.15	0.10	0.06

x	2.0	Assume first and last point lie on line of best fit.	5.1
$\log_{10} y$	-0.28		-1.22



Find gradient.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-0.28 - (-1.22)}{2 - 5.1}$$

$$= \frac{0.94}{-3.1} = -0.30 \text{ (2d.p.)}$$

Equation of Straight Line

$$Y = mX + c$$

$$\log_{10} y = -0.30\,x + c$$

Find y intercept

(2.0 , -0.28) lies on the line.

$-0.28 = -0.30 \times 2 + c$

$-0.28 + 0.6 = c$

$c = 0.32$

$$\log_{10} y = -0.30\,x + 0.32$$

handheld gives intercept as 0.33

Equation of Exponential Function

$$\log_{10} y = -0.30\,x + 0.32$$

$\log_{10}(?) = -0.30$ $\quad (?) = 10^{-0.3}$

$\log_{10}(?) = 0.32$ $\quad (?) = 10^{0.32}$

$$\log_{10} y = \log_{10}(10^{-0.3})\,x + \log_{10}(10^{0.32})$$

$$\log_{10} y = \log_{10}(0.5)\,x + \log_{10}(2.1)$$

$$\log_{10} y = x \log_{10}(0.5) + \log_{10}(2.1)$$

$$\log_{10} y = \log_{10}(0.5)^{x} + \log_{10}(2.1)$$

$$\log_{10} y = \log_{10}((0.5)^{x} \times 2.1)$$

$$y = 2.1\,(0.5)^{x}$$